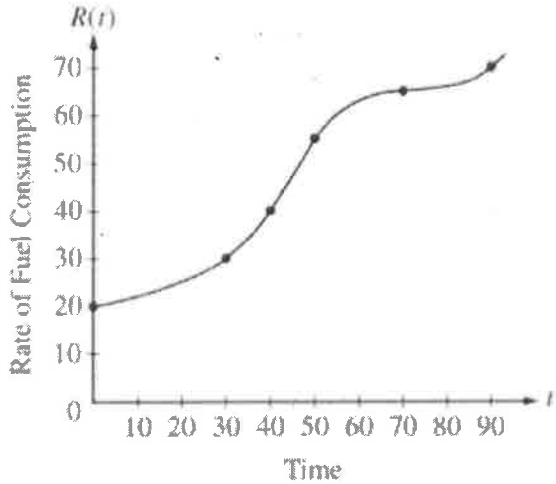
AP Review Worksheet #4
AB 2003 Free Response Questions

Name: Key



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

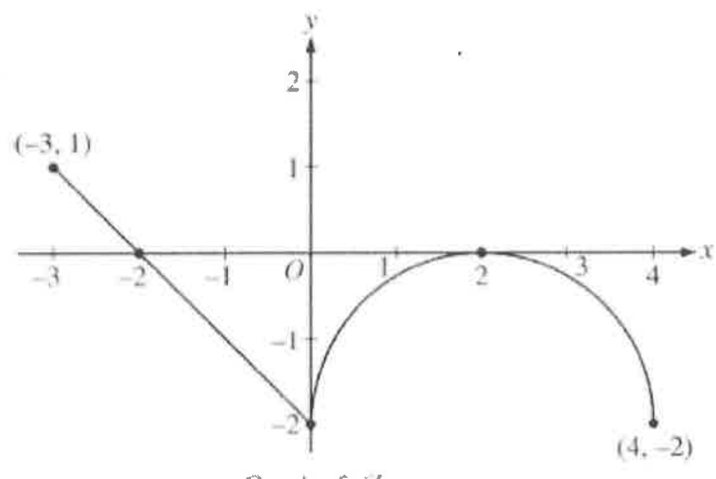
   (a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

   $\approx$ slope @ $t = 45 \rightarrow \frac{R(50) - R(40)}{50 - 40} = \frac{55-40}{10} = \frac{3}{2}$ $\frac{\text{gal}}{\text{min}^2}$

   (b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

   $= 0$

   because Max of R' @ $t = 45$



Graph of f'

4. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

   (a) On what intervals, if any, is f increasing? Justify your answer. f' + on $[-3, -2)$

   (b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

   where f' goes from incr to decr or decr. to incr. @ $x = 0, 2$

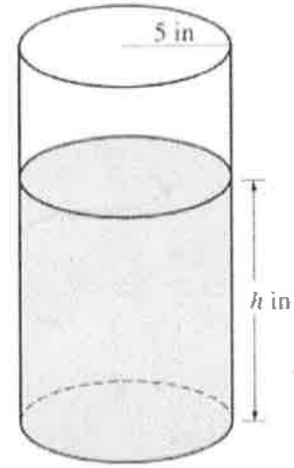
   pointy place ok

   f'' undef. @ $x = 0$

   (c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

   $f'(0) = -2$

   $y = -2x + 3$



5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.) $\longrightarrow V = 25\pi h$

Given; $\frac{dV}{dt} = -5\pi\sqrt{h}\ \frac{in^3}{sec}$

(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

$\frac{dV}{dt} = 25\pi\frac{dh}{dt} \longrightarrow \frac{-5\pi\sqrt{h}}{25\pi} = \frac{25\pi}{25\pi}\frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

$\sqrt{3+1} \stackrel{?}{=} 5-3$

$2 = 2$ ✓ yes; pieces connect @ $(3,2)$

★ Full points awarded by equating the limits

$\lim_{x\to 3^-}\sqrt{x+1} \stackrel{?}{=} \lim_{x\to 3^+}(5-x)$

$\sqrt{3+1} = 5-3$

$2 = 2$ ✓

Therefore $\lim_{x\to 3} f(x) = 2$

(c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?

Means: g is continuous

so: $k\sqrt{3+1} = m(3) + 2$

$2k = 3m + 2$

Need a 2nd relationship in k and m:

Derivatives from left & rt at $x = 3$ must be =

$g'(x) = \begin{cases} \frac{k}{2}(x+1)^{-1/2} & ; (0,3) \\ m & ; (3,5) \end{cases}$

$\lim_{x\to 3^-} g'(x) = \lim_{x\to 3^+} g'(x)$

$\frac{k}{2\sqrt{3+1}} = m$

$k = 4m$

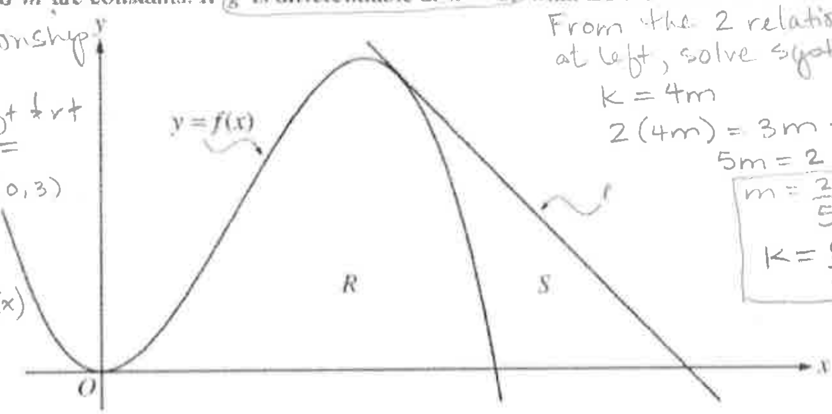
From the 2 relationships at left, solve system:

$k = 4m$

$2(4m) = 3m + 2$

$5m = 2$

$\boxed{m = \frac{2}{5} \quad k = \frac{8}{5}}$



1. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

$y - 9 = -3(x - 3)$

$y = -3x + 18$ ✓

(a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

$f'(x) = 8x - 3x^2$

$f(3) = 36 - 27 = 9 \quad (3, 9)$

$f'(3) = 24 - 27 \rightarrow -3$ slope

6. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

(a) Find $f''(3)$.

$f''(x) = x \cdot \frac{1}{2}[f(x)]^{-1/2} f'(x) + (1)[f(x)]^{1/2}$

$f''(x) = \frac{x \cdot x\sqrt{f(x)}}{2\sqrt{f(x)}} + \sqrt{f(x)}$

$\rightarrow f''(x) = \frac{x^2}{2} + \sqrt{f(x)}$

$f''(3) = \frac{9}{2} + \sqrt{25}$

$\boxed{= 19/2}$